647510 **MSC1MatC101x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Nov/Dec-2023
Algebra 1(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. Figures to the right indicate marks.
2. All questions are compulsory.

Que.1(A) Answer any TWO of the following questions. (10)

1) Show that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.
2) Prove that every group of order p^2 is abelian, where p is a prime.
3) State and prove First Isomorphism Theorem of groups.

Que.1(B) Answer any ONE of the following questions. (04)

1) State and prove Cayley's theorem.
2) Let H and K be two subgroups of a group G. Let $HK = \{hk | h \in H, k \in K\}$. Prove that HK is a subgroup of G if and only if $HK = KH$.

Que.2(A) Answer any TWO of the following questions. (10)

1) If H is a subgroup of a finite group G and $|H|$ is a power of a prime p, then show that H is contained in some Sylow p-subgroup of G.
2) Prove that group of order 56 is not simple.
3) State and prove first Sylow theorem.

Que.2(B) Answer any ONE of the following questions. (04)

1) Let G be a finite abelian group and a prime p divides o(G). Let P be a sylow p-subgroup of G. Prove that P is the only sylow p-subgroup of G if and only if P is a normal subgroup of G.
2) Let G be a finite group and p be a prime such that $p^m | \, O(G)$ but $p^{m+1} \nmid O(G)$. Let n_p be the number of $sylow - p - subgroups$ of G. Then prove that $n_p | O(G)$.

Que.3(A) Answer any TWO of the following questions. (10)

1) State and prove Fundamental Theorem of Ring theory.
2) State and prove Second Isomorphism Theorem of ring homomorphism.
3) Prove that the only homomorphisms from the ring of integers $\mathbb{Z}$ to $\mathbb{Z}$ are the identity and zero mappings.

Que.3(B) Answer any ONE of the following questions. (04)

1) Define the following terms:
   a). Ring b). Integral Domain c). Prime Ideal d). Maximal Ideal
2) Prove that in a nonzero commutative ring with unity, an ideal M is maximal ideal if and only if R/M is a field.

Que.4(A) Answer any TWO of the following questions. (10)

1) Let $f(x) \in \mathbb{Z}[x]$ be primitive polynomial. Then prove that $f(x)$ is reducible over $\mathbb{Q}$ if and only if $f(x)$ is reducible over $\mathbb{Z}$.
2) State and prove Division Algorithm Theorem.
3) Show that Every Euclidean domain is a principal ideal domain.

Que.4(B) Answer any ONE of the following questions. (04)

1) Define Irreducible Polynomial and prove that $x^3 - 5x + 10$ is irreducible over $\mathbb{Q}$.
2) Prove that product of two primitive polynomial is again primitive polynomial.

Que.5(A) Answer any TWO of the following questions. (10)

1) Determine all groups of order 99.
2) State and prove Eisenstein Criterion.
3) Construct a field of order 25.

Que.5(B) Answer any ONE of the following questions. (04)

1) Define Subgroup and Normal Subgroup of a Group. Show that group of order 108 has a normal subgroup of order 27.
2) Define field with an example. Prove that a finite integral domain is a field.

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